SIMPLE METHOD OF LIDAR POINT DENSITY DEFINITION FOR AUTOMATIC BUILDING RECOGNITION
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Abstract. Aerial laser scanning is a modern and accurate remote sensing technology how to scan the earth’s surface and to get its digital surface model. The digital surface model is applied for different economical tasks. The result of aerial laser scanning is 3D point cloud, which must be preprocessed before usage. There are three groups of preprocessing tasks: noise filtering, object recognition and generation of vector maps or 3D model. This report is related with the object recognition field. The main parameter of aerial laser scanning is the point density, which is expressed as the point number per square meter. Therefore, it is important to know the minimal point density per square meter, which must be satisfied to recognize the object for stakeholders and the delivery of LiDAR data. The existing scientific publications only describe recognition methods, but they do not provide some precise method to chose the necessary point density for business needs. So, there is the need for some method, which can be used to define this minimal point density. This document provides the simple equation to calculate the minimal point density for building recognition. The equation is expressed from the analysis of the mathematical model. The analysis is based on the exploration of the object location patterns and probability to detect this object. The theoretical model is experimentally evaluated using high density LiDAR data, the point density minimization algorithm and the building recognition method.

Keywords: building, LiDAR, point density, recognition.

Introduction
People constantly change the environment adapting it according to their cultural, business and consumer needs, transforming the environment under the concept of their world view. These concepts and values have been constantly changing throughout the human history.

The modern world requires timely, detailed and objective analysis of all these changes more than ever before to prevent the harm of a man or his living environment. The modern technologies allow to automate the immediate control over the human activities using remote sensing technologies and computers.

Remote sensing is the research method of objects without a physical contact with them. Search tasks and territorial monitoring are the main use cases of remote sensing. The civil remote sensing is used for the following purposes:

- to search natural and energy resources;
- to find missing people after natural or technological hazards;
- to monitor land cover and land use;
- to monitor and protect the natural environment;
- to monitor natural processes;
- to control and monitor manned or unmanned vehicles;
- to monitor water, ground, aerial and space objects.

But the remote sensing only provides the source of information as the image of a current spatial situation for analysis with the following conclusions from it.

It is necessary to interpret the image of geospatial information to evaluate the geospatial situation, to forecast its development and to prepare the plan of actions according to the assigned task. Data interpretation is the process of data classification, which fulfils nameless digital data with the semantic and attributive components.

The simplest method of data classification is a manual method, but it is very inefficient, if the monitoring territory is large regions, states or the whole surface of the Earth, and it is especially inappropriate, if the monitoring must be done in real time. To solve the tasks of such scale, automatic and semi-automatic recognition systems are a more appropriate choice.
Computer vision is the field of science, which researches and develops the methods to recognize depicted objects in image using computers. Computer vision together with remote sensing is used to solve different business problems.

- The scientific paper [1] describes the complex semi-automatic system of forest inventory, which uses as raw data: aerofoto, spectral and LiDAR data to recognize the types of trees and to assess their geometrical sizes.
- The scientific papers [2-6] describe methods, which can be applied to recognize urban objects like buildings and vegetation.
- The scientific paper [7] describes the method, which recognizes the types of agricultural lands using multispectral data.
- The scientific paper [8] describes a simple statistical method to recognize the types of land cover using ortophoto.

One of remote sensing methods is terrestrial laser scanning, which is used to produce the digital 3D model of terrain. This work is related with the terrestrial laser scanning and the problem of automatic building recognition. If the existing scientific publications only describe recognition methods, they do not open some simple method to define the necessary point density for the recognition method. The goal of this research is to get a simple method, which can be applied to define the necessary point density of laser scanning to detect and recognize man-made structures.

Materials and methods

The first stage is to evaluate the minimal point density to recognize the building in 3D point cloud using theoretical analysis, which is based on the analysis of laser scanning features and the exploration of object location patterns and probability to detect this object.

The second stage is to verify the theoretical analysis and to evaluate the minimal point density experimentally minimizing the point density of the sample.

Mathematical model and theoretical analysis

The airborne laser scanning is the type of remote sensing to get the digital 3D model of the Earth surface. A laser scanner works as a rangefinder: knowing the coordinates of a scanning device, scan direction and the distance to the object, it is possible to calculate the coordinates of each object point (Fig. 1).

![Fig. 1. Scheme of airborne laser scanning](image)

Two types of ranging principles are usually applied by LiDAR [9-11]:

- pulsed ranging;
- or continuous wave (CW).

The formula (1) is used to calculate the distance to an object in the case of a pulse laser:

\[ d = \frac{ct}{2}, \]  

where \( c \) – speed of light; 
\( t \) – round trip travel time.

The formula (2) is used to calculate the distance to an object in the case of CW-laser:
\[
d = \frac{1}{4\pi} c \varphi T.
\]

where \( \varphi \) – phase shift;
\( T \) – period of the signal.

A laser scanner has the following parameters:

- a pulse repetition frequency;
- the field of view;
- the maximal scan range;
- a laser wavelength;
- the maximal number of recorded multiple returns;
- the type of LiDAR mechanical part, which creates the trajectory of scanning.

The pulse repetition frequency is the number of emitted pulses over the given period of time. The higher pulse repetition frequency of the laser scanning gives the higher density of object points per the ground sample.

The distance between two adjacent points on the scan line and the swath width depend on the field of view (FOV) and the flying height. The **swath width** is calculated by the formula (3) [10; 11]:

\[
w = 2h \cdot tg(\alpha),
\]

where \( w \) – swath width;
\( h \) – flying height;
\( \alpha \) – half of the field of view.

A scan pattern depends on the scanning mechanism (Fig. 3) [9], [12]:

- a sinusoidal scan pattern in the case of an oscillating mirror;
- parallel scan lines in the case of a rotating polygon;
- an elliptical scan pattern in the case of a nutating mirror.

![Scan patterns](image)

**Fig. 3. Scan patterns:** a – sinusoidal scan pattern; b – parallel scan lines; c – elliptical scan pattern

The granularity of laser scanning is determined by the density of points on the object surface. But the high density of points is not always the indicator of the laser scanning quality. The multiple returns from an emitted impulse is the typical feature of forests that is reflected in the high density of points, however, these multiple returns from a single impulse are only a noise for the tasks of building search.

The minimal density of points per ground sample is the usual requirement of the laser scanning specification. A ground sample distance (GSD) is the length of a square side, into which the study region is broken on. The average density of points per the whole region is not a correct requirement for laser scanning, because some ground samples (GS) can contain low density, but others – high, which together satisfy the average density.

If a search target is buildings, the useful points are only the single and last returns, therefore other points must be ignored estimating the usable point density (4):

\[
\rho = \frac{|P_s| + |P_l|}{N \cdot g^2},
\]

where \( P_s \) – array of single returns;
\( P_l \) – array of last returns;
An object must be detected and recorded in an image, before it can be recognized. If a study region with buildings is projected on the grid of ground samples (Fig. 4), it is possible to see, that all ground samples can be grouped into three categories:

- which belong to the building;
- which partially belong to the building;
- which do not belong to the building.

One point per GS is a sufficient point density to detect the part of the object, which takes the whole GS. So, a building with sufficient geometrical sizes, which takes one whole GS, can be detected using the point density equal to one point per GS, where the point density is calculated by the formula (4). The building with the size equal to \(2\sqrt{2}g \times 2\sqrt{2}g\), where \(g\) – ground sample distance, is the object with the sufficient geometrical sizes. This size is defined analysing the worst location of the building, when the center of the building is located on the cross of lines of GS grid with a turn 90 degrees assuming that a laser beam always falls out the object (Fig. 5). If the point density is equal to \(1 \text{ p·m}^{-2}\), the buildings with the sizes \(2.8 \times 2.8 \text{ m}^2\) are all detected.

Fig. 4. Building projected on grid of ground samples

Fig. 5. Building with sufficient geometrical sizes to be detected by one point per GS

Only roofs of buildings are visible parts for airborne laser scanning, therefore the features of roofs are used to recognize buildings. There is the list of features, which are used to identify a roof [2-6]:

- the high elevation of points (the borders of buildings);
- the plane formed by the adjacent points;
- the high echo-rate (the low transparency);
- the reflectance of materials.

The high elevation is used to identify the attention points for the following segmentation. The echo-ratio and the reflectance of materials are used as the parameters of a segment.

One object point and one object GS is not sufficient information to recognize an object, because they can belong to the trunk of a tree, electric pole, wall and to other objects. So, the geometrical form of the segment (plane) is important information.

Every plane is described by three points. It is possible to see, that all points of LiDAR are placed in the line on the roof (Fig. 6), therefore, three object GS must be used to describe the shape of an object and it is minimal information for recognition.

According to Fig. 5, the man-made structure of size \(2\sqrt{2}g \times 2\sqrt{2}g\) is detectable by the sufficient number of points for recognition. So, the formula (5) can be used for rough assessment of ground sample distance:

\[
g_{\text{min}} = \sqrt{\frac{S_{\text{min}}}{8}}
\tag{5}
\]

where \(g_{\text{min}}\) – minimal GSD to detect the building with the area greater than \(S_{\text{min}}\)

\(S_{\text{min}}\) – minimal area of the building

If the formula (5) is defined for the ideal conditions, the formula (6) has the correction coefficient, which is got experimentally:
\[ g_{\text{min}} = \eta \cdot \sqrt[8]{S_{\text{min}}} \]  \hspace{1cm} (6)

where \( g_{\text{min}} \) – minimal GSD to recognize the building with the area greater than \( S_{\text{min}} \)
\( S_{\text{min}} \) – minimal area of the building
\( \eta \) – correction coefficient for recognition.

Now, the case of a man-made structure, which is sufficiently less than GS, is analyzed. The structure of size 0.25g x 0.25g can be considered as a sufficiently small structure, because this structure can be undetected, if LiDAR trace goes through the center of GS by vertical, horizontal or diagonal, and the increase of the points does not help (Fig. 7). If one assumes, that the earth is flat as in Fig. 1, the location of every LiDAR point can be calculated, but the different factors as the relief of the surface, air flows and the human factor of the pilot add the randomness into this systematic location of points (Fig. 6).

Fig. 6. Roof and trace of LiDAR points  \hspace{1cm} Fig. 7. Sufficiently small man-made structure

If it is assumed that the trace of points is predefined, the location of a sufficiently small structure is random within GS, consequently the chance to detect this structure is calculated using the probability theory (7):
\[ p(n) = 1 - \left(1 - \frac{S}{g^2}\right)^n \] \hspace{1cm} (7)

where \( n \) – number of points
\( S \) – area of the structure
\( g \) – GSD

According to Fig. 8, the probability to detect the sufficiently small structure is very low, as a result the probability of recognition is noticeably lower, and that does not strongly differ from the previously told.

Fig. 8. Probability to detect a sufficiently small man-made structure

So, the man-made structures, which are proportional to GS, must comply with the systematic and random distribution of points together.

The different cases with proportional structure are depicted in Fig. 9.
The structure $g \times g$ is detectable with the probability 100% in the case of Fig. 9a, but only one GS is detected, but it does not satisfy the condition of recognition (three GS).

The case of Fig. 9e is characterized by the probability to detect the structure and by unsatisfied condition for recognition.

In the cases of Fig. 9b-d, the probability to detect the structure is lower than 100%, but there is the probability to satisfy the recognition condition, too.

The next rules of probability are defined for the next assumptions:

- If the laser scanning point falls into the corner of the building, it has the probability 0.25 to deviate into the direction of the building;
- If the laser scanning point falls into the border of the building, it has the probability 0.5 to deviate into the direction of the building;
- If the laser scanning point falls into a corner or a border out, it has not a sufficient potential to deviate into the direction of the building.

Example of calculation:

Case: one point per GS, average case with vertical scanning trace (Fig. 9b).

To detect: $D = 4 \times 0.25 = 1$, where 4 points and 0.25 is the value of the corner case.

To recognize is equal to detect three points: $R = P_4 + P_3 = 0.25^4 + 4 \times (0.25^3) \times (1 - 0.25) \approx 0.05$, where $P_4$ is the probability to detect four points, $P_3$ – to detect three points from four points.

The formula (8) is used to calculate the step between the points:

$$dx = d / n \quad (8)$$

where $n$ – number of points

$d$ – length of laser scanning trace in GS

The accuracy of building borders is an important factor for the client, which is defined by the detected small details of a building border. The percent of the detected details can be evaluated by the formula (7) from the point of view of randomness (Fig. 11), but the formula (8) can be applied for systematic approach (Fig. 12).
### Table 1

<table>
<thead>
<tr>
<th>Point density per GS</th>
<th>Best case (Fig. 9a)</th>
<th>Average case, vertical (Fig. 9b)</th>
<th>Average case, diagonal (Fig. 9c)</th>
<th>Worst case, vertical (Fig. 9d)</th>
<th>Worst case, diagonal (Fig. 9e)</th>
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\(D\) — probability to detect the object, \(R\) — probability to recognize the object (to detect three object points)

**Fig. 11.** Probability to detect parts depending on their size

**Fig. 12.** Length of parts, which are detectable by appropriate point density
Results and discussion

Two experiments have been completed to evaluate the theoretical model. The used sample has the following parameters:

Area: 17.7 ha
Buildings: 16.4 %
High Vegetation: 23.6 %

1st experiment

Goal: to compare the point density of a multireturn case with the sample filtered by the last return.

Results: the mean point density decreased from 18.43 to 12.79 p·m\(^{-2}\) (Fig. 13-14).

2nd experiment

Theory: the stratified random method was used to decrease the point density.

Goal: to view the dependency between the point density and recognition accuracy

Raw data: laser point cloud filtered by the last return.

Recognition algorithm: energy minimization approach without filter by area [2].

Results: statistical recognition results depend on the GSD, but the shape precision depends on the GSD and the point number (Fig. 15-17). The selected analyzed building has the area equal to 104 m\(^2\) and the shape close to theoretical – square form with a turn 90 degrees. According to formula (5), the minimal GSD must be 3.6 m to detect the building what is experimentally proved (see Fig. 16).
The authors of [13] recommend the point density, which is above five points per square meter. According to the mathematical model (Table 1, Fig. 11-12), the optimal point number can be in the range [3; 5] points per GS. The experiments (see Fig. 17) have proved the selected range. So, the recommended point density is above three points, that is a smaller number than the recommended range by other authors [13].

The increase of the point number does not increase the recognition precision for the rastered method, the main factor is scanning resolution – GSD (Fig. 15) that coincides with other authors’ results [13].

The authors of [13] recommend the GSD smaller than 0.5 m. According to [14], the point density of one point per 5 m² (GSD = 2.2 m) is sufficient for relief detection. So, the optimal GSD is in the range [0.5; 2.2]. The visual investigation shows (Fig. 16), that GSD = 1 is more close to the waiting shape, then the correction coefficient η of the used method is equal to 1/3 for the formula (6).

Conclusions

The research has shown that the point number must be above three points per GS to identify the building shape.

The minimal GSD to recognize a building can be calculated by formula (6), but the research was done using only one parameter – the area. According to the fact that most building recognition methods use the height difference between ground samples, the research must be continued considering the building height.
The results can be checked using other recognition methods to generalize the results. The used building recognition method belongs to the raster type. The point cloud recognition methods can be sensitive to the point number too and have other optimal ranges that must be mathematically and experimentally checked.

Acknowledgment
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References
13. Tomljenovic I., Rousell A. Influence of point cloud density on the results of automated Object-Based building extraction from ALS data. Proceedings of the AGILE’2014 International Conference on Geographic Information Science, Castellón, Spain, June 3-6, 2014.