MATHEMATICAL MODEL FOR DETERMINATION OF LOSSES OF SUGAR BEARING-MASS WHEN SUGAR BEET TOPS ARE REMOVED

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Abstract. Production of sugar from sugar beets is an important branch of agriculture and food industry in many European countries. In spite of the great progress, the contemporary root crop top removing machines cause considerable losses of the sugar-bearing mass and do not meet the agrotechnical requirements. When the tops are removed without a top feeler, a part of the root crop is cut off together with the tops. The removal quality of sugar beet tops during harvesting affects the amount of losses of sugar. A model has been developed for the determination of the losses of the sugar-bearing mass depending on the statistical characteristics of the height of elevation of the sugar beet heads from the ground and the density of the root crops. If the values of the statistical characteristics of height elevation of the root crop heads and their density are extreme, the losses of the sugar-bearing mass exceed the agrotechnical requirements two times.

Key words: sugar beet, top removal, losses.

Introduction

Production of sugar beet in the European countries is an important branch of agriculture. The greatest output of sugar is in France (9 million t), Russia, Ukraine, Germany, Poland and the Netherlands. A great part of the produced beets is lately used for extraction of bioethanol.

A sufficiently detailed analysis of the designs of the operating tools for the top removal is made in the works by L.B.Pogorely, V.M.Bulgakov, M.V.Tatynko, etc. [1-7]. However, there are no designs of the top removing machines for the present day which would completely meet the agrotechnical requirements for the losses of the sugar-bearing mass.

The removal quality of sugar beet tops during harvesting affects the amount of losses of sugar. For instance, increased heap pollution of the sugar root crops by 1 % lowers the sugar output by 0.1 %, but if the root crops are kept in piles with a 4 % content of leaves, then the average daily losses of sugar are 0.5 % [6; 7]. The agrotechnical requirements determine that the leaf remnants on the root crop heads should not exceed 1.5 % of the total mass of the root crops but the losses of the sugar-bearing mass not more than 2 %. In spite of the great progress, the contemporary root crop top removing machines cause considerable losses of the sugar-bearing mass and do not meet these agrotechnical requirements. Therefore, today an urgent task is to find such technical solutions which would allow avoiding these losses.

In contemporary top removers the root crop tops are predominately removed without a feeler. When the tops are removed without a feeler, a part of the root crop is cut off together with the leaves. The amount of these losses depends on the cutting height, and in practice it often exceeds the agrotechnical requirements [8-12].

The aim of the work is to work out a mathematical model for determination of the losses of the sugar-bearing mass when the sugar beet tops are removed.

Materials and methods

The research was carried out by using the methods of mathematical modelling on the basis of the probability theory. Considering the fact that it is not possible to determine an integral expression obtained by modelling for the appearance probability of a subinterval of the elevation heights of the root crop heads in quadratures, it was solved by using numerical integration according to Simpson’s rule (prismoidal formula).

It has been established by the results of independent research [13] that the share of root crops from which the leaves were cut off by the root crop top removing machines operating without a feeler was, on the average, 2 % if the cutting height of the leaves was 2 cm; if the cutting height was high – 26.6; if it was low – 7.5, but the share of the root crops cut in a satisfactory manner – 60.1 %. Such
statistics is a witness of the losses of the sugar-bearing mass and significant remnants of the leaves on the root crops. These results apply to an average operating velocity of travel of the machine 6 km/h, which is a low velocity of travel, and its choice is determined by a desire to reach the quality of work of the passive operating tools for preliminary cutting of the tops. The digging tools can work at a significantly greater speed than the top removing ones. In [2] it has been found that the basic factor affecting the losses of the sugar-bearing mass when working without a feeler is distribution of the elevation heights of the sugar beet heads from the level of the ground, particularly, its statistical characteristics: mathematical expectation and root-mean-square deviation. It is also known that the indicated factors may vary within wide limits and differ for different sugar beet crops. Generally these factors depend on the sugar beet growing agrotechnics, and the soil and weather conditions. Besides the statistical characteristics of distribution of the elevation heights of the root crop heads, the amount of the losses of the sugar-bearing mass depends also on the quantity of the root crops per hectare. The quantity of root crops per hectare (the density of crops) may vary within the limits from 50 to 100 thousand plants per hectare [3]. Therefore, it is very important to study the degree of a complex impact of the indicated factors upon the losses of the sugar-bearing mass.

For this purpose we will use the familiar parametric model of the root crop head and the leaves [4]. The dependencies of the parameters of the root crop head and the leaves have the following appearance:

\[
\begin{align*}
    h_3 &= a \cdot h + b; \\
    d_1 &= m \cdot h + n; \\
    d_3 &= d_1 + 2 \cdot h_3 \cdot \tan \alpha 
\end{align*}
\]

where
- \( h_3 \) – distance from the upper part of the head to the bottom of the green leaves;
- \( h \) – elevation height of the head;
- \( d_1 \) – diameter of the upper part of the root crop plant;
- \( d_3 \) – diameter of the bunch of leaves;
- \( \alpha \) – half-angle of the conicity of the head;
- \( a, b, m, n \) – constant values [5].

The losses of the sugar-bearing mass and the remnants of leaves for the root crops \( (M_i) \) of the given subinterval of the elevation heights are determined according to formula:

\[
M_i = F(h_i; h_z) \cdot P(h_i; h_{i+1}) \cdot N, \tag{2}
\]

where
- \( F(h_i; h_z) = V_k \cdot \rho \) – losses of the sugar-bearing mass or the remnants of the plant tops for a single root crop;
- \( h_i \) and \( h_z \) – average elevation height of a subinterval of elevation heights and the cutting height of the root crop head (the cut-off part of the root crop is measured from its upper part);
- \( V_k \) and \( \rho \) – respective volume and density of the root crop and the leaves;
- \( N \) – quantity of root crops of the given interval per unit of area (pcs·ha\(^{-1}\));
- \( P(h_i; h_{i+1}) \) – appearance probability of a subinterval of the elevation heights of the root crop heads determined according to formula

\[
P \left( h_i; h_{i+1} \right) = \frac{1}{\sigma \sqrt{2\pi}} \int_{h_i}^{h_{i+1}} \exp \left\{ -\frac{(h - m)^2}{2\sigma^2} \right\} dh, \tag{3}
\]

where
- \( \sigma \) – roof-mean-square deviation of the elevation height of the root crop heads at a normal distribution law;
- \( m \) – mathematical expectation of the elevation height of the root crop heads at a normal distribution law.
Results and Discussions

The integral in expression 3 is not determined in quadratures, therefore the appearance probability of the root crops of the given interval of the elevation heights is determined by numerical integration using Simpson’s rule (prismoidal formula). After summing up the remnants of the leaves and the losses of the sugar-bearing mass for all intervals of the elevation heights we will obtain the summary mass of the remaining leaves on the root crops \( B \) and the summary losses of the sugar-bearing mass \( GM \) (a part of the root crop heads cut off together with the top leaves) per unit of area:

\[
B = \sum_{i=a}^{b} \left[ N \cdot F_i \left( \frac{h_i + h_{i+1}}{2} ; h_i \right) \cdot \left( \frac{h_{i+1} - h_i}{3m} \sum_{j=0}^{m} c_j \cdot f(h) \right) \right], \tag{4}
\]

\[
GM = \sum_{i=a}^{b} \left[ N \cdot F_i \left( \frac{h_i + h_{i+1}}{2} ; h_i \right) \cdot \left( \frac{h_{i+1} - h_i}{3m} \sum_{j=0}^{m} c_j \cdot f(h) \right) \right], \tag{5}
\]

where \( m \) – quantity of subintervals into which the entire range of the elevation heights of the root crop heads is divided: \( m = 2U; U=1,2,3,4,...; \)

\( c_j \) – coefficient at the values of the subintegral function in the corresponding points (a coefficient of the polynomial used in numerical integration by the Simpson’s method).[14].

The losses of the sugar-bearing mass and the remnants of the top leaves on the root crops have been calculated depending on the roof-mean-square deviation and density of the plants per one hectare. For this purpose an algorithm was applied, a programme created in the MS Excel 2010 programming environment, and graphs of the indicated dependencies were built (Fig. 1). Calculation of the losses of the sugar-bearing mass was executed in the 60-80 mm operative range of the cutting heights of the root crop heads without using a feeler [3].

On the basis of graphic dependencies reflected in Figure 1, it is possible to assert that the losses of the sugar-bearing mass increase sharply when both factors – the roof-mean-square deviation and density of the plants per one hectare – are increased. It should be mentioned that in the case of a lower limit of the operative range of the cutting heights of the root crop heads without using a feeler (Fig. 1 b, d) the agrotechnical requirements are completely fulfilled as regards the losses of the sugar-bearing mass. But in this case one should expect increased heap pollution of the sugar root crops, and therefore here it would be purposeful to lower automatically the indicated cutting height. In the case of the upper limit of the cutting heights of the root crop heads without using a feeler there exists an impermissible zone set by the agrotechnical requirements for the losses of the sugar-bearing mass (more than 2 % of the total mass of the root crops) (Fig. 1 a, b), and in this case it is appropriate to increase the cutting height of the root crop heads without using a feeler in order to avoid the losses of the sugar-bearing mass.

It was established by the previous research that the statistical characteristics of the elevation heights of the root crop heads may vary within the limits: mathematical expectation \( m = 40...60 \) mm, and the roof-mean-square deviation \( \sigma = 15...25 \) mm [5]. Extreme values of these characteristics are their maximal values at which, according to [5], the losses of the sugar-bearing mass and the remnants of leaves on the heads of the root crops increase to 5-6 %.

The agrotechnical requirements for the losses of the sugar-bearing mass when the tops are removed without a top feeler can be met only if the cutting height is increased. But then the requirements are not satisfied regarding the amount of the leaves remaining on the root crops. Therefore, considering the obtained results of the research in the impact of the factors mentioned (i.e. a 4-times increase in the losses of the sugar-bearing mass in the case of lower cutting of the root crop heads, and a 2-times increase in the case of higher cutting of the heads) it is purposeful to develop an automatic regulation system of the cutting height of the root crop heads. The functions of such a system are: constant monitoring of the dimensional characteristics of the root crops, determination of their statistical characteristics, and uninterrupted in time optimisation of the cutting height in order to avoid excessive losses of the sugar-bearing mass. Application of an automatic regulation system of the
cutting height of the root crop heads may lower the heap pollution of the root crops not less than by 2-5%.

Fig. 1. **Dependencies of the losses of the sugar-bearing mass** $B$ **at the cutting height 60 mm** ($a$) **and 80 mm** ($b$) **without using a feeler; two-dimensional sections of the indicated dependencies at the cutting height 60 mm** ($c$) **and 80 mm** ($d$) **without using a feeler**

**Conclusions**

1. The developed calculation model indicates that the losses of the sugar-bearing mass depend on the statistical characteristics of the elevation height of the root crop heads and the density of the root crops.
2. When the values of the statistical characteristics of the elevation heights of the root crop heads and their density are extreme, the losses of the sugar-bearing mass exceed the agrotechnical requirements two times.
3. In order to improve the quality indicators of top removal of the root crops, it is purposeful to create an automatic regulation system of the cutting height of the root crop heads.

**References**